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Abstract for Form 298

A multidimensional periodicity detection algorithm is established, and implementation has been made on ECoG data and with broadband and 1/f noise. The algorithm requires the characterization of periodic continuous wavelet transforms, and the solution of an optimization problem coupled with an averaging method for mixed-norm spaces.

A transition of this technology has been made to Techno-Sciences, Inc. in their NASA supported work for Health Maintenance of jet engines using eddy current sensors.

A multidimensional irregular sampling technique is also established, and the corresponding algorithm solves signal reconstruction problems in MRI for sampled spectral data on interleaving spirals related to fast imaging techniques. The tools required are our results from the theory of frames, as well as the Beurling-Landau analysis of balayage.

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New Techniques in Signal and Image Processing Based on the Uncertainty
    Principle, Irregular Sampling, and Finite Fields
AFOSR Grant F49620-96-1-0193

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1. Technology Transfer

    Because of our periodicity detection and multidimensional irregular sampling algorithms, and because of the relevance of the latter algorithm to unsettled questions about MRI and SAR, we have made contact with several organizations with the goal of productive technology transfer. These organizations and our contacts within them are

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13. ABSTRACT (Maximum 200 words)

A multidimensional periodicity detection algorithm is established, and implementation has been made on ECoG data and with broadband and 1/f noise. The algorithm requires the characterization of periodic continuous wavelet transforms, and the solution of an optimization problem coupled with an averaging method for mixed-norm spaces.

A transition of this technology has been made to Techno-Sciences, Inc. in their NASA supported work for Health Maintenance of jet engines using eddy current sensors.

A multidimensional irregular sampling technique is also established, and the corresponding algorithm solves signal reconstruction problems in MRI for sampled spectral data on interleaving spirals related to fast imaging techniques. The tools required are our results from the theory of frames, as well as the Beurling-Landau analysis of balayage.

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DARPA, Dr. D. Healy, 703-696-0143
VEXCEL Corp., Dr. C. Johnston, 301-583-0273
      (Dr. Johnston was formerly a consultant on this contract)
USC-ISI, Dr. P. Topiwala, 703-812-3708.

A substantive transition has been made with TECHNO-SCIENCES, INC.,
which was awarded a NASA contract based in part on the wavelet periodicity
detection algorithm developed by us with this AFOSR grant. In conjunction with
their Phase 1 award, MIPS (Maryland Industrial Partnerships) made
a supplementary award to us.

The basic problem is Health Maintenance of jet engine blades
by means of ECSs (Eddy Current Sensors). This is a technology based
on detecting changes in magnetic flux. The engine design is due to
Pratt-Whitney, and the ECS design is due in part to Lucent Technologies
and General Dynamics. We are involved in the signal processing component
of this research, dealing with optimal numbers of sensors and sensor
placement in order to quantify in real time any type of blade vibration
or blade deterioration. Our wavelet periodicity detection algorithm
plays a natural role in solving essential vibration detection problems.

2. Objectives

Our project aimed to solve certain problems of signal reconstruction
by designing fast algorithms with noise reduction capability, generally
in the context of turbulent behavior, irregularly spaced data, and a
multidimensional setting. The most immediate applications are MRI and SAR.

We addressed these
problems with our results on frames, with the Beurling-Landau point of
view on balayage, and with an analysis of piecewise constant wavelets,
combined with optimization and averaging methods in geometrical linear
algebra.

The major goal was the development of software for
computationally effective algorithms based on multidimensional irregular
sampling and periodicity detection theorems.

Applications were envisaged on multidimensional
irregular sampling problems associated with the control of smart structures,
with SAR systems, and with topics such as MRI and gridding reconstruction.
Also, besides our ongoing work with epileptic seizure prediction, our
periodicity detection algorithm is naturally suited to address problems in
cockpit motion-sickness, vibration identification, fault detection, and
water wave prediction.

In general, our objectives were met mathematically. Our software
objectives are incomplete because of fundamental stability problems.
Our applications to MRI were successful by our development of a
fast imaging method for signal reconstruction from spectral data
on interleaving spirals. In stead of addressing control of smart
structures, we became involved with signal processing problems
associated with Health Maintenance of jet engine blades by means of
eddy current sensors. This is promising on-going work.

3. Status of Effort

The status of our effort is divided into three
parts: signal reconstruction for multidimensional irregularly
spaced sampling, periodicity detection, and computer implementation of
our irregular sampling algorithm. The Appendix comprising Section 8
of this report describes results developed in the early part of Period 2,
and these results form the background for Sections 3 and 4.

Hui-Chuan Wu and I have used theorems of Beurling and Henry Landau to
prove new results relating frames and balayage. These results are then used
in conjunction with various notions of density to prove new sampling formulas
for 2-dimensional irregularly spaced data which include data on spirals.
In particular, we can constructively and
iteratively choose points on a given spiral so that the Fourier frame
reconstructions from all sampled values at these points give rise to
a Paley-Wiener space of fixed bandwidth. We can measure the bandwidth for
the signals in this space; and, more importantly, if we are given a
fixed bandwidth, then we can construct interleaving spirals so that our
iterative method of choosing sampling points give rise to all finite energy
signals in that band by means of Fourier frame reconstructions. This
algorithm solves a basic problem in MRI.
The same method can be used for points in annular segments, and,
consequently, our goal is to implement this algorithm in SAR reconstruction.
Error analysis, quantitative comparisons, and implementation with real
data have yet to be made. Our method avoids
all regridding and interpolation, and so, in theory at least, there is
less complexity in our approach than in some existing methods.

Because of the periodic structure of seizure data (Section 8) and
the low available sampling rates in the EEG-ECoG data available to us,
Goetz Pfander and I were led to over-sampling methods and the theory
of piecewise constant wavelets in devising an effective periodicity
detection algorithm, which also incorporates compatible averaging and
optimization methods.
In particular, if a known periodicity is to be detected in a noisy
environment, our method constructs a specific wavelet, and operates on an
associated wavelet transform in such a way as to produce the desired
periodicity. The algorithm is based on a theorem, and simulations,
which we knew would be successful because of the theorem, are presented
in items 4 and 6 of Section 6.
We have extended this algorithm to higher dimensions. We
have also proven that continuous periodic wavelet transforms (properly
discretized in terms of frames) play an essential role in periodicity
detection. Finally, we have characterized the wavelets giving rise
to periodic wavelet transforms in terms of our original piecewise
constant wavelets and a logarithmic term. We have not yet determined the
role of the logarithmic term in the signal processing problems we are
addressing.

Our objectives concerning fast algorithms have reached the following
state of affairs. Melissa Harrison has developed the software to implement
the irregular sampling theory of Benedetto and Heller. In fact, her
implementation goes beyond this latter theory, and we have developed

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a number of techniques to make the transition from continuous to finite in terms of Gram operator approximations for the case of Fourier frames.

4. Accomplishments

To reach the status reported in the previous section, we have accomplished the following.

The multidimensional irregular sampling sign reconstruction formulas for MRI, that Hui-Chuan Wu and I proved and that were mentioned in Section 3, depend on the following background and results relating to frames and balayage.

Balayage is a notion from potential theory which allows the decomposition of a point mass spread out to larger sets. Beurling formulated the notion in terms of measures and their Fourier transforms, and there is an equivalent formulation relating finite energy and sup norms. The basic theorem, where Henry Landau has also made a considerable contribution, is the following. Let S be a uniformly discrete set of sampling points in Euclidean space, and let E be a sufficiently thick set of spectral synthesis; then balayage is possible for S and E if and only if S is the set of sampling points for a Fourier frame for the Paley-Wiener space having frequency band E. There are natural results between upper and lower densities of S and the measure of E in the case of balayage. The deepest of these results is due to Landau, although some of our results also play a role.

Such results, combined with the previous theorem, give rise to examples of sampling formulas for irregularly spaced sets of the type we have already mentioned. In fact, our major accomplishment is a proof of the following theorem: if E is a convex, compact set of Euclidean space with respect to translations by a given uniformly discrete set S, then the elements of S are the frequencies for a Fourier frame for the finite energy signals on a precisely measured polar set of E. This measure allows us to calculate computable frame bounds; and these, in turn, allow us to solve the MRI signal reconstruction problem.

Goetz Pfander and I developed a theory of piecewise constant wavelets to maximize the usefulness of preseizure data in predicting periodic behavior in EEG seizure data, see Section 8. During Period 2, we realized that our approach could be adapted to deal with a host of periodicity detection problems.

We have constructed a periodicity detection algorithm, and as mentioned in Section 3, it incorporates piecewise constant wavelets in conjunction with averaging and optimization methods. These methods involve constructing periodicity dependent wavelets, and proving wavelet inequalities by means of geometrical and linear algebra arguments. The implementation of our algorithm requires an analysis of periodic continuous wavelet transforms.

Mathematically, the continuous wavelet transform of T-periodic functions for a piecewise constant wavelet of degree M is T-periodic in time and MT-periodic in scale. We detect lattice patterns in time-scale space in terms of relative maxima of such wavelet transforms. This is done for a given T-periodic function by constructing an optimal piecewise constant wavelet, as mentioned in

the previous paragraph. The optimality is determined by computing certain orthogonal projections onto hyperplanes associated with wavelets. The periodicity detection is accomplished by wavelet averaging, which simultaneously reduces broadband noise in a manner analogous to but different from the thresholding methods we introduced in our Wavelet Auditory Model.

Melissa Harrison has extended her MATLAB software package from Period 1 (where she implemented the one-dimensional Benedetto-Heller irregular sampling theory) to a sophisticated user friendly package for general Fourier frames. Our mathematical results underlying the general package involve a new analysis of pseudo-inverses, an iterative method for Gram operator computations, an eigenvalue study for dealing with SVDs, and proofs of convergence theorems for the finite approximations required in computer implementation and estimation of infinite dimensional inverse frame operators. The code and our simulations are available.

5. Personnel Supported

John J. Benedetto, PI
Melissa L. Harrison, Graduate Student
Hui-Chuan Wu, Graduate Student
Goetz Pfander, Graduate Student
Sherry Scott, Graduate Student
Songkiat Sumetkijakan, Graduate Student

Notes
1. This support is exclusive of consultancy support for Carolyn Johnston during Period 1 which overlapped with our original Period 2.
2. Melissa L. Harrison and Hui-Chuan Wu received their Ph.D. degrees in May 1998, and Goetz Pfander received his Ph.D. in August 1999. Naturally, they acknowledged AFOSR support in their theses.

6. Technical Publications by J.J. Benedetto (acknowledging AFOSR support for this contract)

Journal Publications

1. Generalized harmonic analysis and Gabor and wavelet systems, Invited Paper in the AMS Proc. of Symposia in Applied Mathematics, Wiener Centenary Congress, 52 (1997), 85-113.

2. The theory of multiresolution analysis frames and applications to filter banks (with S. Li), Applied and Computational Harmonic Analysis, 5 (1998), 389-427.

3. Sampling multipliers and the Poisson Summation Formula (with G. Zimmermann), Journal of Fourier Analysis and Applications, 3 (1997), 505-523.

4. Wavelet detection of periodic behavior in EEG and ECoG data (with G. Pfander), Invited Paper at IMACS, Berlin, 1997, pages 75-80.

5. Self-similar pyramidal structures and signal reconstruction
   (with M. Leon and S. Saliani), SPIE, Wavelet Applications V,
   3391 (1998), 304-314.

6. Wavelet periodicity detection algorithms (with G. Pfander),
   SPIE, Wavelet Applications in Signal and Image Processing VI,
   3458 (1998), 48-55.

7. Balayage and multidimensional signal reconstruction from data
   on curves (with H.-C. Wu), in preparation.

8. The theory of local sampling (with N. Atreas and C. Karanikas),
   to be submitted.

9. The construction of single wavelets in d-dimensions (with M.
   Leon), J. Geometric Analysis, accepted.

10. Wavelet periodicity detection: theory and applications (with
    G. Pfander), in preparation.

11. The construction of multiple dyadic minimally supported
    frequency wavelets on R^d (with M. Leon), AMS Contemporary Math.
    Series, accepted.

12. A Beurling covering theorem and multidimensional irregular
    sampling (with H.-C. Wu), SampTA'99, Loen, Norway,
    1999, invited.

13. A multidimensional irregular sampling algorithm and
    applications (with H.-C. Wu), IEEE - ICASSP, Phoenix, Special
    Session on Recent Advances in Sampling Theory and Applications,
    4 (1999), 4 pages, invited.

Books

14. Harmonic Analysis and Applications, CRC Press, Inc.,
    Boca Raton, FL, 1997.

15. Modern Sampling Theory (with P. Ferreira, ed.), Birkhauser
    Boston, 1999 publishing date.

Invited Book Chapters

16. Frame signal processing applied to bioelectric data, in
    Wavelets in Medicine and Biology (A. Aldroubi and M. Unser,
    editors), CRC Press, Inc., Boca Raton, 1996, pages 493-512.

17. Noise reduction in terms of the theory of frames, in
    Signal and Image Representation in Combined Spaces (J. Zeevi and
    R. Coifman, editors), Academic Press, New York, 1997, pages
    259-284.

18. Frames, sampling, and seizure prediction, in
    Hong Kong Wavelet Workshop (K.-S. Lau, editor), Springer-Verlag,
    New York, 1998, pages 1-25.

19. Gabor systems and the Balian-Low Theorem (with C. Heil and D. Walnut), in Gabor Analysis and Algorithms, Theory and Applications, H. G. Feichtinger and T. Strohmer, editors, Birkhauser, Boston, 1998, pages 85-122.

20. Sampling theory and wavelets, NATO-ASI, in Signal Processing for Multimedia 1998, J. Byrnes, editor, Kluwer Publishers, The Netherlands, 1998.

21. Frames, irregular sampling, and a wavelet auditory model, (with S. Scott), in Sampling Theory and Practice, F. Marvasti, editor, Plenum Publishers, 2000.

22. Wavelet frames: multiresolution analysis and extension principles (with O. Treiber), in Wavelet Transforms and Time-Frequency Signal Analysis, L. Debnath, editor, Birkhauser, 2000.

Due to time considerations, I was unable to accept several invitations for articles.

7. Conference and Colloquia Invited Presentations by J.J. Benedetto for 1997-1999 (acknowledging AFOSR support for this contract)

AMS (San Diego) Wavelet Special Session, January 1997
Brown University Colloquium, February 1997
Johns Hopkins University Colloquium, March 1997
Plenary Lectures at Hong Kong Wavelet Workshop, May 1997
Plenary Lecture at SampTA97, in Aveiro, Portugal, June 1997
IMACS (Berlin), Pfander spoke for me, August 1997
University of Toledo, Shoemaker Lecture Series, November 1997
AMS (Baltimore) Wavelet Special Session, January 1998
Plenary Lecture at International Conference on Spectral Methods in Medical Signal Processing (Munich), March 1998
SPIE (Orlando), Leon spoke for me, April 1998
Math Awareness Week at U.MD. - Math/Stat, April 1998
University of Thessaloniki, June 1998
Plenary Lectures at NATO-ASI (Il Ciocco), Signal Processing for Multimedia, July 1998
SPIE (San Diego), Goetz Pfander spoke for me, July 1998
University of Buenes Aires, 3 symposia and a 6 lecture ASI, October 1998
MAA (MD-DC-VA at Towson U.) featured speaker, November 1998
AMS (San Antonio) Wavelet Special Session, January 1999
IEEE-ICASSP (Phoenix) Sampling Special Session, 1999
Canadian Mathematics Society (St. John's), May 1999
University of Illinois - Mathematics in Science and Society Colloquium, April 1999
University of Illinois, analysis seminar, April 1999
SampTA International Symposium on Sampling Theory (Trondheim, Norway) plenary speaker, August 1999
SampTA International Symposium on Sampling Theory (Trondheim, Norway) special session, August 1999.
Defense Applications of Signal Processing - US/Australia (DASP99), August 1999.

Due to time considerations, I was unable to accept several speaking invitations.

8. Appendix

Our initial objectives concerning noise reduction were achieved at the theoretical level in the beginning of Period 2 for the case of quantization noise. This involved our theory of frame multiresolution analysis (FMRA) with Shidong Li. These results are related to Sigma-Delta AD Converters, and one of our goals for Period 3 was to establish this relationship. The effectiveness of our FMRA theory, which surprisingly implements ideal perfect reconstruction narrow band filters, has been tested on MRI data, and the results are very good.

Our other effort in noise reduction concerned work with noisy EEG data vis a vis the more precise but invasive ECoG method. This is part of ongoing work conducted with Goetz Pfander, and it involves our analysis of seizure prediction. which, in turn, has led to our approach in periodicity detection.

Our first effort on quantitative multidimensional irregular sampling was reported in the Period 1 Progess Report. It dealt with Kluvanek's theorem, lattice theory, and Albert Cohen's notion of congruence which he used to characterize orthonormal sets of integer translates in terms of quadrature mirror filters. We obtained interesting sampling formulas in this way, but we were not able to deal with certain basic problems arising in applications, e.g., the characterization of spirals and sampling sets on spirals in order to obtain sampling formulas of the type required in MRI data analysis. As noted in Section 2, we have overcome this obstacle. Further, we have shown that congruence can be used in a quantitative way in dealing with the construction of single multidimensional orthonormal wavelets and in reducing sampling rates for subsets of frequencies in a given large bandwidth.

The balayage relationship between finite energy signals and the sup norm was mentioned in Section 4 with regard to our irregular sampling results. There is a relationship between this form of balayage and peak sets arising in antenna theory and having to do with ultraflat polynomials. The analysis of this relationship is part of our continuing research

From the point of view of solving the basic epilepsy seizure prediction problem, we have provided a mathematical model based on the following three steps.

a. ECoG data of an individual patient are analyzed through spectral and wavelet methods to extract periodic patterns associated with epileptic seizures of a specific patient.

b. Using this knowledge of seizure periodicity, we construct an optimal piecewise constant wavelet designed to detect the epileptic periodic patterns of the patient.

c. We introduce a fast discretized version of the continuous wavelet transform, as well as wavelet averaging and optimization techniques, to detect occurence and period of the seizure periodicities in the preseizure EEG data of the patient.

Our primary objectives concerning seizure prediction algorithms have been met. The specific problem was to find out, as far in advance as

possible, when an epileptic seizure will occur. Because of the time series structure of preseizure and seizure data, we have been able to show (using ECoG data) that spectral methods will not generally yield more than a 15 second lead time.

Goetz Pfander has written code to implement our seizure prediction algorithms. The algorithms themselves are necessarily wavelet-based, i.e., the ECoG-EEG data analysis forces a wavelet technology, and they are constructed with a natural noise suppressant feature. Our seizure prediction software has only been tested on synthetic data.